Exercise 19

(a) Show that two parallel planes are either identical or they never intersect.

(b) How do two nonparallel planes intersect?

Solution

Part (a)

Suppose that two planes are parallel. Then they both have the same normal vector $\mathbf{n} = (n_x, n_y, n_z)$. Let $\mathbf{r}_1 = (x_1, y_1, z_1)$ be the position vector for a point in one plane and let $\mathbf{r}_2$ be the position vector for a point in the other plane. The equations for these planes are

$$\begin{aligned} \mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_1) &= 0 & \mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_2) &= 0 \\ (n_x, n_y, n_z) \cdot (x - x_1, y - y_1, z - z_1) &= 0 & (n_x, n_y, n_z) \cdot (x - x_2, y - y_2, z - z_2) &= 0 \\ n_x(x - x_1) + n_y(y - y_1) + n_z(z - z_1) &= 0 & n_x(x - x_2) + n_y(y - y_2) + n_z(z - z_2) &= 0 \\ n_x x - n_x x_1 + n_y y - n_y y_1 + n_z z - n_z z_1 &= 0 & n_x x - n_x x_2 + n_y y - n_y y_2 + n_z z - n_z z_2 &= 0 \\ n_x x + n_y y + n_z z &= n_x x_1 + n_y y_1 + n_z z_1 & n_x x + n_y y + n_z z &= n_x x_2 + n_y y_2 + n_z z_2. \end{aligned}$$

If $(x_1, y_1, z_1) = (x_2, y_2, z_2)$, then the planes are identical. Otherwise, they will never intersect because

$$n_x x_1 + n_y y_1 + n_z z_1 \neq n_x x_2 + n_y y_2 + n_z z_2.$$

Part (b)

The intersection of two nonparallel planes is a straight line.